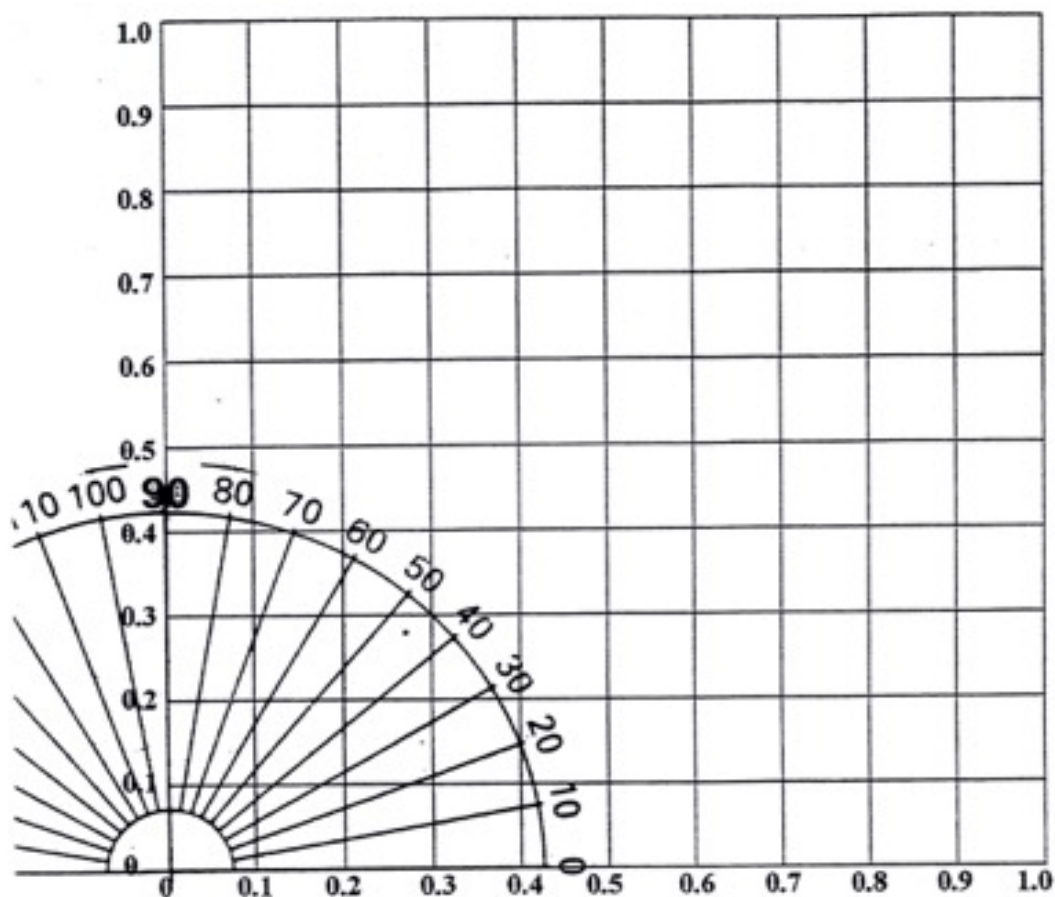# Trigonometry via the Unit Circle and a rotating arm

Students will need access to a rotating arm sheet (below) and will need to fill in the columns two and three of the table on the next page to 2 decimal places

The only knowledge students need to have is: a) a line 1.0 long can be divided into ten equal parts, b) how to read co-ordinates, c) angle is a measure of turn

Before copying the grid below it is important to ensure it is square. Some adjustments may need making. This is because some photocopiers have a habit of enlarging by 2% and if you don't tell it differently, this occurs in either the horizontal or the vertical direction, making the grid no longer 'square', which as far as the following activity goes spells DISASTER!

Students need to draw lines through the marked angles up to 90^0 which is 1.0 long, this as an absolute measure will depend upon how long 1.0 measures.

Table 1

Students can fill in this grid by reading off the co-ordinates of the end of the rotating arm to 2 places of decimal.

Angle	x-ordinate	y-ordinate
0°		
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		
90°		

Once students have completed the task of calculating the x and y-ordinates for all angles from 0° to 90° (going up in 10°), they can use their information to answer the following questions:

The questions below are not intended to be written directly onto a single worksheet. They can be given verbally to students as they complete the tasks, or written on separate A5 sheets and distributed if a verbal explanation is inappropriate.

1. From your results write two or three comments about what you notice.

2. a) At what angle will the x- and y-ordinates be equal?
   b) What are the co-ordinates of this point?

3. Estimate the x-ordinates and y-ordinates for angles rotated by the following degrees of turn (anticlockwise from 0°)

a) 63°, b) 27°, c) 34°, d) 56° e) write about what you notice.

4. Estimate what the angle would be for:
   a) an *x*-ordinate of 0.61, b) an *x*-ordinate of 0.82, c) a *y*-ordinate of 0.56

5. Estimate both *x* and *y*-ordinates for the following degrees of turn:
   110°, b) 150°, c) 200°, d) 300°, e) 400°

6. Using your scientific calculator, check your answers by pressing
   cos (angle) for the *x*-ordinate and sin (angle) for the *y*-ordinate.
   Write your answers in the fourth and fifth columns of the original table.

Table 2

Angle	*x*-ordinate measured	*y*-ordinate measured	Check (Qu 6) *x*-ordinate	Check (Qu 6) *y*-ordinate
0°				
10°				
20°				
30°				
40°				
50°				
60°				
70°				
80°				
90°				

7. What are the boundaries that define whether angles create *x* and *y* co-ordinates as follows: a) (^+x, ^+y), b) (^-x, ^+y), c) (^-x, ^-y), d) (^+x, ^-y),

8. Suppose the rotating arm was length 2 what will happen to the *x*-ordinates and the *y*-ordinates?

At some point students will need introducing to the idea that the rotating arm together with the distances which are measured out by the x- and y-ordinates are in fact right-angled triangles. Students will also need introducing to the vocabulary of sides which are 'opposite' and 'adjacent' to an angle, as well as what the hypotenuse is and where it is situated in a right-angled triangle.

9. In order to complete the table below you need to carry out certain calculations for each:

a) make a sketch, and
b) write out each necessary calculation in full:

Calculate missing lengths to two places of decimal and missing angles to the nearest whole number of degrees.

The intention here is that students draw a sketch diagram and write a full calculation (i.e. which buttons they press on their calculators) for each line of the table below.

Angle	Length of rotating arm or hypotenuse	Length of x-ordinate or adjacent side (cos)	Length of y-ordinate or opposite side (sin)	Pythagoras check
37°	2.00			
48°	3.00			
56°	2.50			
17°	2.25			
75°	6.75			
66°	12.35			
	1.00	0.47		
	2.00	0.94		
	3.00		1.50	
	13.00		5.00	
		1.50	2.00	

Producing sine and cosine graphs
Use the first and the fourth column of the information from table 2 to draw the graph of angle against y-ordinate. Use a range from 0° to 360° on the horizontal axis and a sensible range on the vertical axis.

On the same pair of axes, draw the graph of angle against x-ordinate.