Accessing trigonometry with a rotating arm in a unit circle

Students will need access to a rotating arm sheet and will need to fill in the columns two and three of table 1, below, to 2 decimal place.

They do this by drawing lines each of which are 1 unit long from (0,0) and through the 10º, 20º... up to 80º, then reading the ordinates of the end points of each line.

Also most photocopiers have a habit of enlarging by 2% and if you don't tell it (the photocopier!) differently, this occurs in either in the horizontal or the vertical direction, making the grid no longer 'square'. In terms of the ensuing activity this would be something of a disaster

Before copying the sheet, therefore, it is important to ensure the first copy of the grid squares are in fact square. Adjustments may need making to the copier if this is not the case.

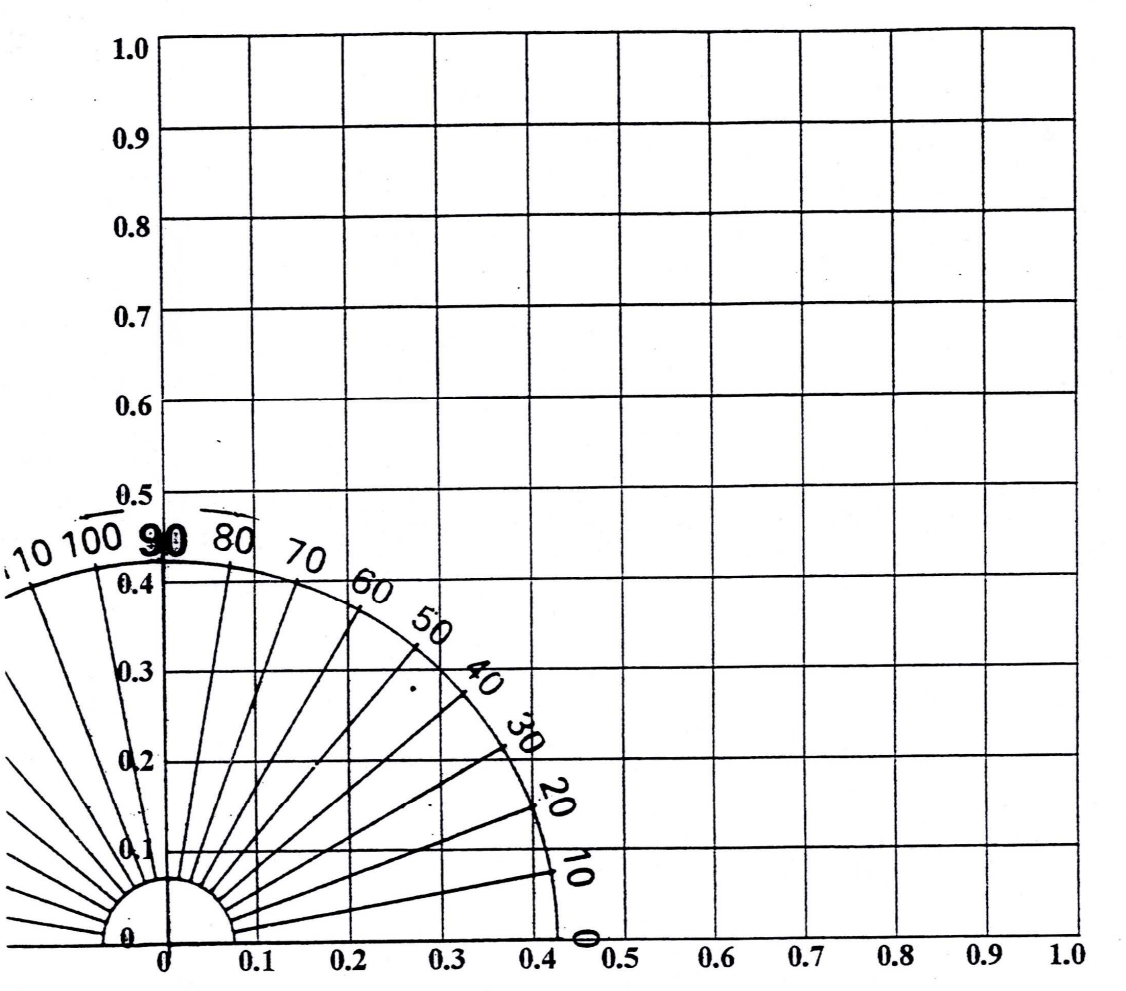

Table 1

Students can fill in this grid by reading off the co-ordinates of the end of the rotating arm to 2 places of decimal.

Angle	***x*-ordinate**	***y*-ordinate**
0°		
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		
90°		

Once students have completed the task of calculating the x and y-ordinates for all angles from 0° to 90° (going up in 10°), th ey can use their information to answer the following questions:

The questions below are not intended to be written as a single worksheet. This is because the examples in questions 3, 4 and 5 are relatively arbitrary; the key issue for students is to analyse what is happening. Questions could be given verbally to students as they complete the task, or even written on separate pieces of paper.

1. From your results write two or three comments about what you notice.

2. a) At what angle will the *x*- and *y*-ordinates be equal?
   b) What are the co-ordinates of this point?

3. Estimate the *x*-ordinates and *y*-ordinates for angles rotated by the following degrees of turn (anticlockwise from 0°)
   a) 63°, b) 27°, c) 34°, d) 56° e) write about what you notice.

4. Estimate what the angle would be for:
   a) an *x*-ordinate of 0.61, b) an *x*-ordinate of 0.82, c) a *y*-ordinate of 0.56

5. Estimate both *x* and *y*-ordinates for the following degrees of turn:
   110°, b) 150°, c) 200°, d) 300°, e) 400°

6. Using your scientific calculator, check your answers by pressing cos (angle) for the *x*-ordinate and sin (angle) for the *y*-ordinate. Write your answers in the fourth and fifth columns of the original table.

Table 2

Angle	Initial *x*-ordinate	Initial *y*-ordinate	Calculator *x*-ordinate	Calculator *y*-ordinate
0°				
10°				
20°				
30°				
40°				
50°				
60°				
70°				
80°				
90°				

Question 7 might be an extension for some students

7. What are the boundaries that define whether angles create *x* and *y* co-ordinates as follows: a) (^{+}x, ^{+}y), b) (^{-}x, ^{+}y), c) (^{-}x, ^{-}y), d) (^{+}x, ^{-}y),

This next aspect of the work could begin with a whole class introduction

Suppose the rotating arm was length 2 what will happen to the *x*-ordinates and the *y*-ordinates?

At some point students will need introducing to the idea that the rotating arm together with the distances which are measured out by the x- and y-ordinates form right-angled triangles. Students will also need introducing to the vocabulary of sides which are ‘opposite’ and ‘adjacent’ to an angle, as well as what the hypotenuse is and where it is situated in a right-angled triangle.

A further extension task could be for some students to see what happens when they square the x and y-ordinates and sum them, which will obviously provide an answer of 1 (or 0.999...)

In order to complete the table below students need to carry out certain processes for each:

a) make a sketch, and
b) write out each necessary calculation in full:

Missing lengths can be written to two places of decimal and missing angles to the nearest whole number of degrees.

The intention here is that students draw a sketch diagram and write a full calculation (i.e. which buttons they press on their calculators) for each line of the table below.

Angle	Length of rotating arm or hypotenuse	Length of *x*-ordinate or adjacent side (cos)	Length of *y*-ordinate or opposite side (sin)	Pythagoras check
37°	2.00			
48°	3.00			
56°	2.50			
17°	2.25			
75°	6.75			
66°	12.35			
	1.00	0.47		
	2.00	0.94		
	3.00		1.50	
	13.00		5.00	
		1.50	2.00	

Again the task below might be an extension for some students or a whole class introduction. This depends on various factors.

Producing sine and cosine curves

Use the first and the fourth column of the information from table 2 to draw the graph of angle against *y*-ordinate. Use a range from 0°to 360°on the horizontal axis and a sensible range on the vertical axis.

On the same pair of axes, draw the graph of angle against *x*-ordinate.